Math 220: Gauss Jordan elimination summary

We can find the solutions to a linear system in a systematic way by repeating the following steps. The following definition will also be handy:

- **Leading Entry** An entry in the matrix for which all numbers to the left (if any), are zeroes.

Beginning in the top row, perform the following steps until there are no more leading entries, or no more rows. After the last step, move **down** to the next row, and begin again at Step 1.

(**Downward Step**)

1. Looking at the current row and below, identify the leftmost column that has a leading entry. This is called the pivot column. If necessary, swap the current row with another row below it so that there are no entries further left than our leading entry in rows below. The nonzero entry in our row is called the pivot.
2. (Optional) Use elementary row operations to change the pivot to a 1. Usually by scaling or swapping rows.
3. Use the replacement rule to eliminate all entries below the pivot.

After repeating the Downward Steps as many times as possible, your matrix is guaranteed to be in "**echelon form**," which is also called triangular form. At this stage, we have a lot of information about the original system of equations. Depending on the question that you are investigating, you may be able to find your solutions from the row echelon form.

Beginning in the bottom-most nonzero row, perform the following steps until you reach the top row. After the last step, move **up** to the next row and begin again at Step 1.

(**Upward Step**)

1. If the row has a non-zero entry, identify the leading entry. Divide the row by the the leading entry so that the leading entry becomes 1. (we will call a leading entry in a matrix in echelon form a **pivot**)
2. Use the current row and elementary row operations to eliminate all entries above the pivot.

After repeating the Upward Steps as many times as possible, your matrix is in "**reduced row echelon form (RREF)**." At this stage, it is easiest to read off the solution to your system of equations. Also, conveniently, there is **exactly one** reduced row echelon form associated with each matrix